Space-Time Coding and Beamforming Hybrids with Interference Avoidance and Robustness to Imperfect Channel Information

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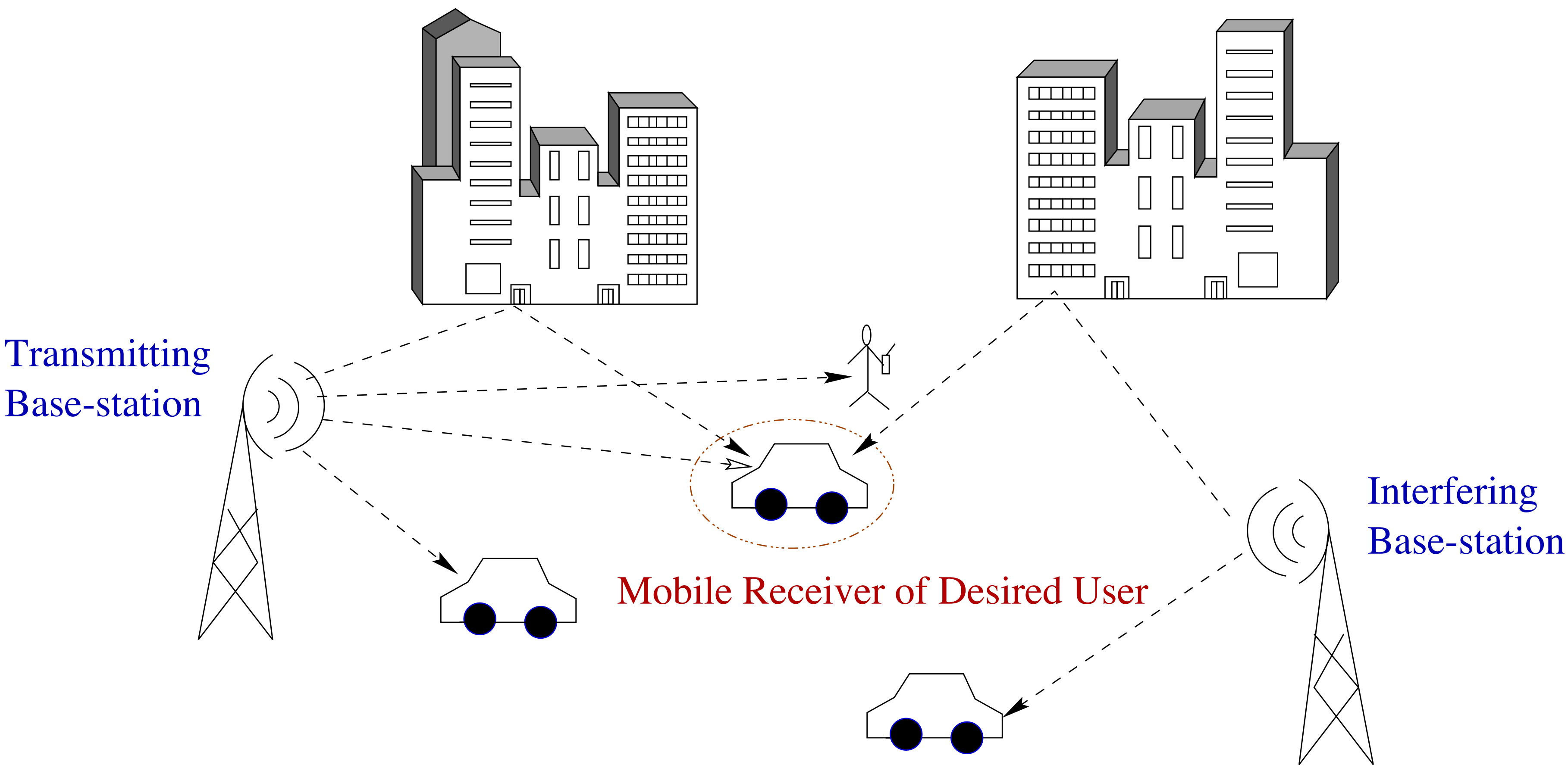
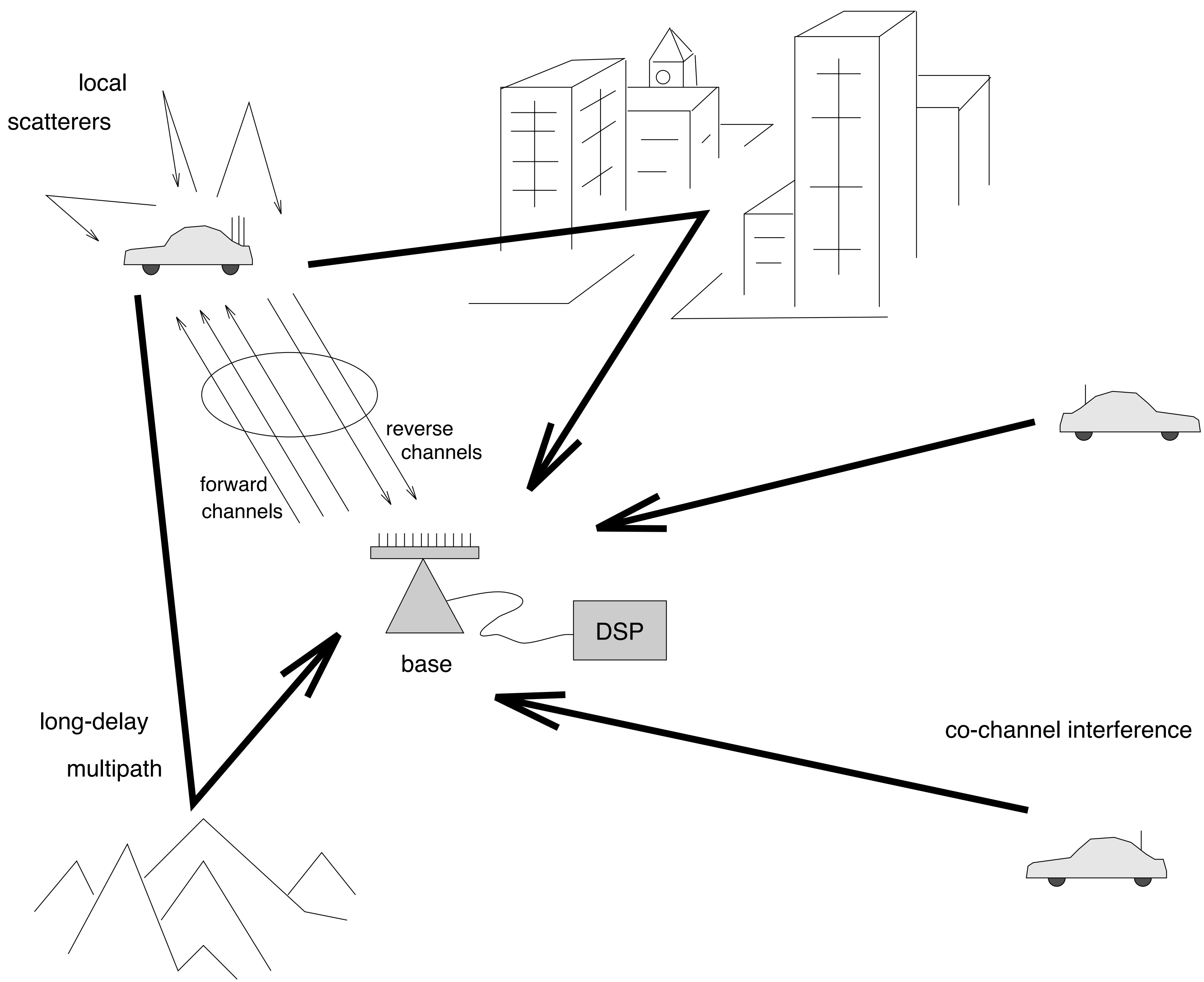
OUTLINE

- SISO Temporal Processing with interference pre-cancelling and “STBC” option
- MISO Space-Time Beamforming with interference pre-cancelling and STBC option
- MIMO Space-Only Beamforming with interference pre-cancelling & STBC option
- MIMO Space-Time Beamforming with interference pre-cancelling and STBC option

Wireless Communications Environment




local
scatterers
reverse
channels
forward
channels
DSP
base
long-delay
multipath
co-channel interference

OUTLINE

- **SISO Temporal Processing with interference pre-cancelling & "STBC" option**
  - development considers single Tx antenna with a single Rx antenna and temporal-only processing at both ends
  - correlation matrices in the development that immediately follows are temporal correlation matrices
- MISO Space-Time Beamforming with interference pre-cancelling and STBC option
- MIMO Space-Only Beamforming with interference pre-cancelling and STBC option
- MIMO Space-Time Beamforming with interference pre-cancelling and STBC option

- SISO data model with temporal-only processing:

$$\mathbf{x}[n] = \sum_{\ell=1}^{K} s_\ell[n]\mathbf{c}_\ell + \boldsymbol{\mathcal{V}}[n]$$

  - $\mathbf{c}_\ell$: $N \times 1$ code vector "carrying" ℓ-th symbol stream
  - $s_\ell[n]$: n-th symbol of ℓ-th information symbol stream
  - $\mathbf{x}[n]$: $N \times 1$ block of received data
  - $\boldsymbol{\mathcal{V}}[n]$: $N \times 1$ block of interference plus noise

- $N \times N$ interference (plus noise) correlation matrix at receiver:

$$\mathbf{R}_{I+N} = \mathcal{E}\{\boldsymbol{\mathcal{V}}[n]\boldsymbol{\mathcal{V}}^H[n]\}$$

- $N \times N$ signal plus interference (plus noise) correlation matrix, assuming independent symbols with $\mathcal{E}\{s_k[n]s_\ell^*[n]\} = \sigma_s^2\delta_{k\ell}$

$$\mathbf{R} = \mathcal{E}\{\mathbf{x}[n]\mathbf{x}^H[n]\} = \sum_{\ell=1}^{K} \sigma_s^2\mathbf{c}_\ell\mathbf{c}_\ell^H + \mathbf{R}_{I+N}$$

- MMSE weight vector employed at receiver to extract k-th information symbol at time n:

$$\mathbf{w}_k = \alpha_k \mathbf{R}^{-1} \mathbf{c}_k$$

  - where $\alpha_k = 1/\mathbf{c}_k^H \mathbf{R}^{-1} \mathbf{c}_k$ so that $\mathbf{w}_k^H \mathbf{c}_k = 1$

- Applying MMSE weight for k-th code to n-th data block

$$\begin{aligned} \mathbf{w}_k^H \mathbf{x}[n] &= \sum_{\ell=1}^{K} s_\ell[n] \mathbf{w}_k^H \mathbf{c}_\ell + \mathbf{w}_k^H \boldsymbol{\nu}[n] && (1) \\ &= \sum_{\ell=1}^{K} \alpha_k s_\ell[n] \mathbf{c}_k^H \mathbf{R}^{-1} \mathbf{c}_\ell + \mathbf{c}_k^H \mathbf{R}^{-1} \boldsymbol{\nu}[n] && (2) \end{aligned}$$

- Designing the codes such that

$$\mathbf{c}_k^H \mathbf{R}^{-1} \mathbf{c}_\ell \propto \delta_{k\ell}$$

  remarkably eliminates "cross-talk" amongst codes while effecting an MMSE receiver

- Conventional Walsh-Hadamard codes do not satisfy this constraint $\Rightarrow$ suffer from "cross-talk" amongst codes as well as interference

- For code-words, use eigenvectors of interference plus noise correlation matrix $\mathbf{R}_{I+N}$

$$\mathbf{R}_{I+N}\mathbf{c}_k = \lambda_k \mathbf{c}_k$$

- Since $\mathbf{R}_{I+N}$ is Hermitian symmetric, it follows that $\mathbf{c}_k$ is also an eigenvector of $\mathbf{R}_{I+N}^{-1}$

$$\mathbf{R}_{I+N}^{-1}\mathbf{c}_k = \frac{1}{\lambda_k}\mathbf{c}_k$$

- Further, since $\mathbf{R}_{I+N}$ and $\mathbf{R}_{I+N}^{-1}$ are both Hermitian symmetric, their eigenvectors are orthonormal (conjugate in the conventional sense)

$$\mathbf{c}_k^H \mathbf{c}_\ell = \delta_{k\ell}$$

- It follows that

$$\mathbf{c}_k^H \{\mathbf{R}_{I+N}^{-1}\mathbf{c}_\ell\} = \frac{1}{\lambda_\ell}\mathbf{c}_k^H \mathbf{c}_\ell = 0 \;\text{ for }\; k \neq \ell$$

- Thus, the eigenvectors of $\mathbf{R}_{I+N}^{-1}$ (same as eigenvectors of $\mathbf{R}_{I+N}$) are conjugate in the conventional sense **and** are $\mathbf{R}_{I+N}^{-1}$-conjugate as well

$$\mathbf{c}_k^H \mathbf{R}_{I+N}^{-1}\mathbf{c}_\ell \propto \delta_{k\ell}$$

- Here we show that selecting codes as eigenvectors of $\mathbf{R}_{I+N}$ implies they are $\mathbf{R}$-conjugate **and** $\mathbf{R}^{-1}$-conjugate, as well as $\mathbf{R}_{I+N}$-conjugate, $\mathbf{R}_{I+N}^{-1}$-conjugate, and $\mathbf{I}$-conjugate (last one means orthogonal in conventional sense)

- Recalling structure of $\mathbf{R}$, we have

$$\begin{aligned}
\mathbf{R}\mathbf{c}_k &= \{\sum_{\ell=1}^{K} \sigma_s^2 \mathbf{c}_\ell \mathbf{c}_\ell^H + \mathbf{R}_{I+N}\}\mathbf{c}_k && (3)\\
&= \sum_{\ell=1}^{K} \sigma_s^2 \mathbf{c}_\ell (\mathbf{c}_\ell^H \mathbf{c}_k) + \mathbf{R}_{I+N}\mathbf{c}_k && (4)\\
&= \sigma_s^2 \mathbf{c}_k + \lambda_k \mathbf{c}_k && (5)\\
&= \gamma_k \mathbf{c}_k && (6)
\end{aligned}$$

  – implies $\mathbf{c}_k$ is an eigenvector of $\mathbf{R}$

- since $\mathbf{R}$ is Hermitian symmetric $\Rightarrow$ $\mathbf{c}_k$ is an eigenvector of $\mathbf{R}^{-1}$

$$\mathbf{R}^{-1}\mathbf{c}_k = \frac{1}{\gamma_k}\mathbf{c}_k$$

- **VIP Result**: selecting the k-th code as an eigenvector of $\mathbf{R}_{I+N}$, it follows that the MMSE weight vector for extracting the k-th symbol is equal to the code itself:

$$\mathbf{w}_k = \frac{\mathbf{R}^{-1}\mathbf{c}_k}{\mathbf{c}_k^H \mathbf{R}^{-1}\mathbf{c}_k} = \frac{\frac{1}{\gamma_k}\mathbf{c}_k}{\frac{1}{\gamma_k}\mathbf{c}_k^H \mathbf{c}_k} = \mathbf{c}_k$$

- since $\mathbf{c}_k$ is an eigenvector of $\mathbf{R}^{-1}$ with unit length

- directly analogous to using Walsh-Hadamard codes in white noise: the MMSE receiver for the k-th Walsh-Hadamard code is the code itself

- Applying $\mathbf{c}_k$ to n-th data block

$$\begin{aligned} \mathbf{w}_k^H \mathbf{x}[n] &= \mathbf{c}_k^H \mathbf{x}[n] && (7) \\ &= \sum_{\ell=1}^{K} s_\ell[n](\mathbf{c}_k^H \mathbf{c}_\ell)\mathbf{c}_\ell^H + \mathbf{c}_k^H \boldsymbol{\nu}[n] && (8) \\ &= s_k[n] + \mathbf{c}_k^H \boldsymbol{\nu}[n] && (9) \end{aligned}$$

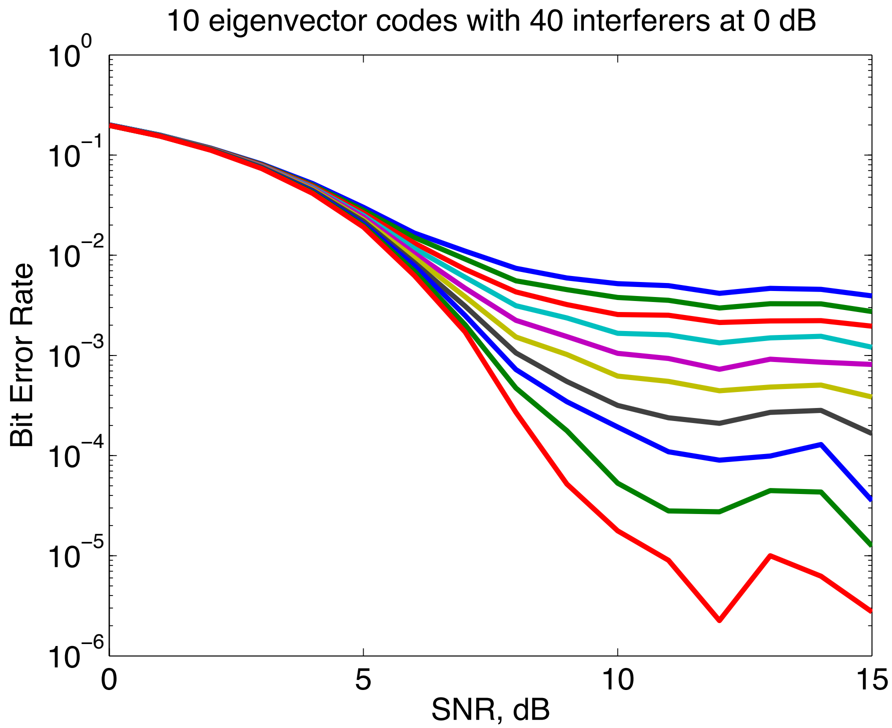
  – since $\mathbf{c}_k$ is also $\mathbf{I}$-conjugate as well

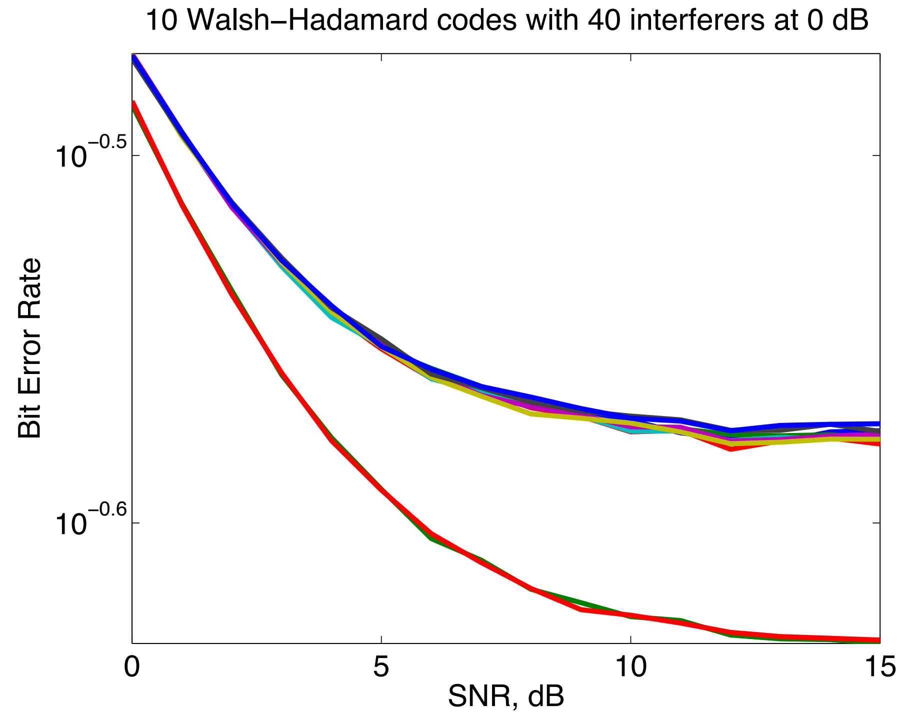
- **Summarizing**: selecting code $\mathbf{c}_k$ as an eigenvector of interference plus noise correlation matrix $\mathbf{R}_{I+N}$, we have shown that
  - $\mathbf{c}_k$, $k = 1, ..., K$ are $\mathbf{I}$-conjugate $\Rightarrow \mathbf{c}_k^H \mathbf{c}_\ell = \delta_{k\ell}$
  - $\mathbf{c}_k$, $k = 1, ..., K$ are $\mathbf{R}_{I+N}$-conjugate $\Rightarrow \mathbf{c}_k^H \mathbf{R}_{I+N} \mathbf{c}_\ell \propto \delta_{k\ell}$
  - $\mathbf{c}_k$, $k = 1, ..., K$ are $\mathbf{R}_{I+N}^{-1}$-conjugate $\Rightarrow \mathbf{c}_k^H \mathbf{R}_{I+N}^{-1} \mathbf{c}_\ell \propto \delta_{k\ell}$
  - $\mathbf{c}_k$, $k = 1, ..., K$ are $\mathbf{R}$-conjugate $\Rightarrow \mathbf{c}_k^H \mathbf{R} \mathbf{c}_\ell \propto \delta_{k\ell}$
  - $\mathbf{c}_k$, $k = 1, ..., K$ are $\mathbf{R}^{-1}$-conjugate $\Rightarrow \mathbf{c}_k^H \mathbf{R}^{-1} \mathbf{c}_\ell \propto \delta_{k\ell}$
- **VIP Result**: selecting code $\mathbf{c}_k$ as an eigenvector of interference plus noise correlation matrix $\mathbf{R}_{I+N}$:
  - MMSE weight for extracting symbol carried by k-th code is equal to the code itself **AND**
  - guarantees no "cross-talk" amongst codes when applying MMSE weight vector to extract symbol carried by k-th code
- **In contrast**: MMSE weight for extracting k-th Walsh-Hadamard code in "colored noise", where $\mathbf{R}_{I+N} \neq \mathbf{I}$, causes "cross-talk" amongst codes

SISO with Temporal-Only Processing

- WLOG constrain "code" words to have unit energy: $\mathbf{c}_k^H \mathbf{c}_k = 1$
- choosing $\mathbf{c}_k$ as an eigenvector of $\mathbf{R}_{I+N}$, it follows from previous development that SNR at receiver at output of k-th matched filter $\mathbf{c}_k$: $\propto \mathbf{c}_k^H \mathbf{R}_{I+N}^{-1} \mathbf{c}_k$
- motivates choosing code "words" as "largest" eigenvectors of $\mathbf{R}_{I+N}^{-1}$
- implies choosing code "words" as "smallest" eigenvectors of $\mathbf{R}_{I+N}$

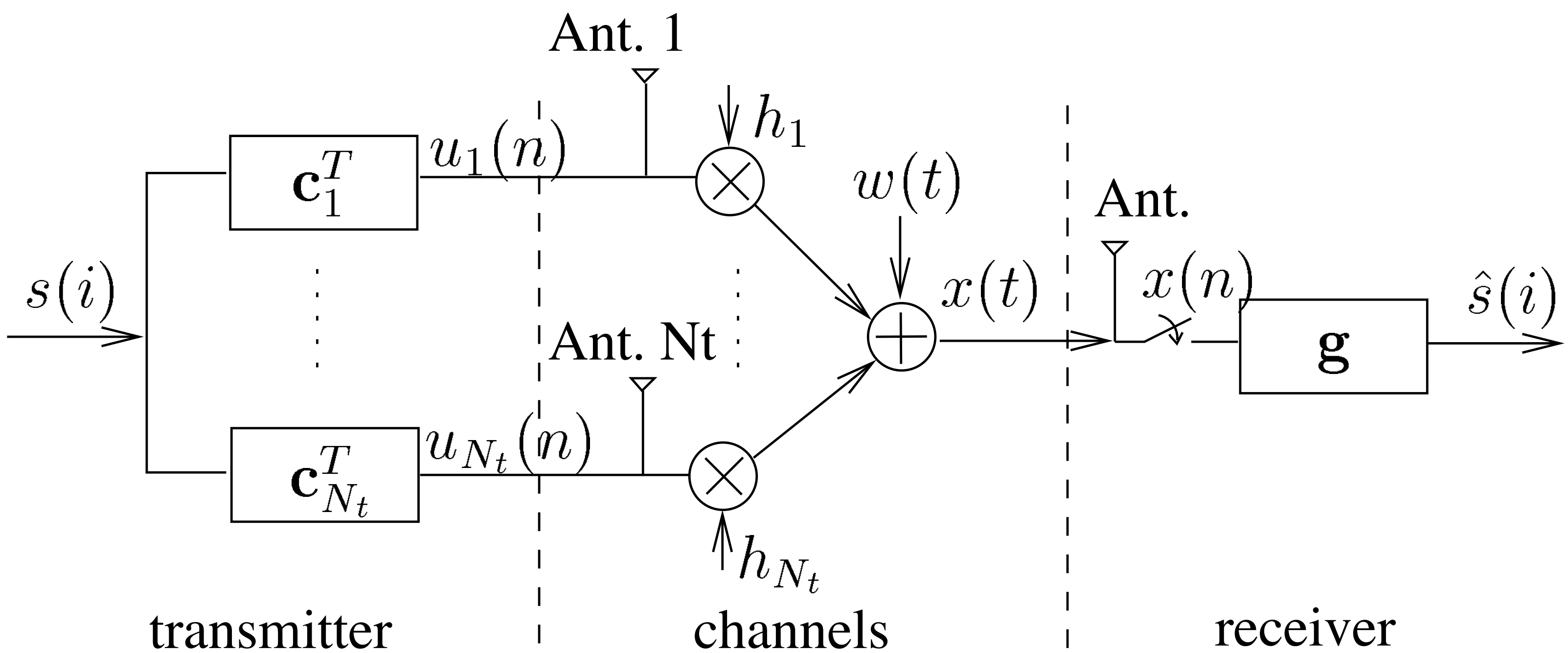
SISO Simulations Incorporating Interference Pre-Cancellation





OUTLINE

- SISO Temporal Processing with interference pre-cancelling and "STBC" option
- **MISO Space-Time Beamforming with interference pre-cancelling & STBC option**
  - development: multiple Tx antenna with single Rx antenna $\Rightarrow$ space-time spreading at Tx, temporal-only processing at Rx
  - temporal correlation matrix of interference fed back from Rx to Tx along with channel gains OR correlation matrix of channel gains
  - joint work with Giannakis & D. Cai at UMN
- MIMO Space-Only Beamforming with interference pre-cancelling and STBC option
- MIMO Space-Time Beamforming with interference pre-cancelling and STBC option



- when transmitting only a single symbol per slot: choose code word at each antenna as "smallest" eigenvector of temporal interference plus noise correlation $\mathbf{R}_{I+N}$ fed back from receiver
- at each "chip" instant, Tx beamforming based on channel gains

- with N_t Tx antennas and single Rx antenna at k-th symbol slot:

$$\mathbf{y} = \sqrt{\mathcal{E}_s}\mathbf{C}\mathbf{h}s + \mathbf{w}$$

- given channel knowledge $\mathbf{h}$, optimal linear receiver is $\mathbf{g} = \mathbf{R}_w^{-1}\mathbf{C}\mathbf{h}$ yielding the output SINR:

$$\gamma = \frac{\mathcal{E}_s}{N_0}\mathbf{h}^{\mathcal{H}}\mathbf{C}^{\mathcal{H}}\mathbf{R}_w^{-1}\mathbf{C}\mathbf{h}.$$

- more realistic to assume knowledge of channel correlation matrix $\mathbf{R}_h$ at transmitter as well as Rx interference plus noise correlation matrix $\mathbf{R}_w$

$$\bar{\gamma} := E[\gamma] = \frac{\mathcal{E}_s}{N_0}E[\mathrm{Tr}(\mathbf{C}^{\mathcal{H}}\mathbf{R}_w^{-1}\mathbf{C}\mathbf{h}\mathbf{h}^{\mathcal{H}})] = \frac{\mathcal{E}_s}{N_0}\mathrm{Tr}(\mathbf{C}^{\mathcal{H}}\mathbf{R}_w^{-1}\mathbf{C}\mathbf{R}_h),$$

- under energy constraint $\mathrm{Tr}(\mathbf{C}\mathbf{C}^{\mathcal{H}}) = 1$, STS matrix: $\mathbf{C}_{opt} = \mathbf{u}_{w,1}\mathbf{u}_{h,1}^{\mathcal{H}}$
$\Rightarrow$ $\mathbf{u}_{h,1}$ is "largest" eigenvector of $\mathbf{R}_h$
$\Rightarrow$ $\mathbf{u}_{w,1}$ is "smallest" eigenvector of $\mathbf{R}_w$

- instead: choose $\mathbf{C}$ to improve SER $\Rightarrow$ power loading
- for tractability, use PEP $\Rightarrow$ if $\mathbf{h}$ is normally distributed, Chernoff bound on unconditional PEP $P(s \to \tilde{s}) := E[P(s \to \tilde{s}|h)]$:

$$P(s \to \tilde{s}) < \left| \mathbf{I}_{N_t} + \frac{\mathcal{E}_s d_{\min}^2}{4N_0} \mathbf{C}^{\mathcal{H}} \mathbf{R}_w^{-1} \mathbf{C} \mathbf{R}_h \right|^{-1} := P_{\text{bound}},$$

- with $\mathbf{C} = \mathbf{U}_w \mathbf{D} \mathbf{U}_h^{\mathcal{H}}$, criterion for diagonal matrix $\mathbf{D}$:

$$\text{maximize} \quad -\log(P_{\text{bound}}) = \sum_{i=1}^{N_{\min}} \log\left(1 + \frac{\mathcal{E}_s d_{\min}^2}{4N_0} \frac{\lambda_{h,i} [\mathbf{D}]_{ii}^2}{\lambda_{w,i}}\right)$$

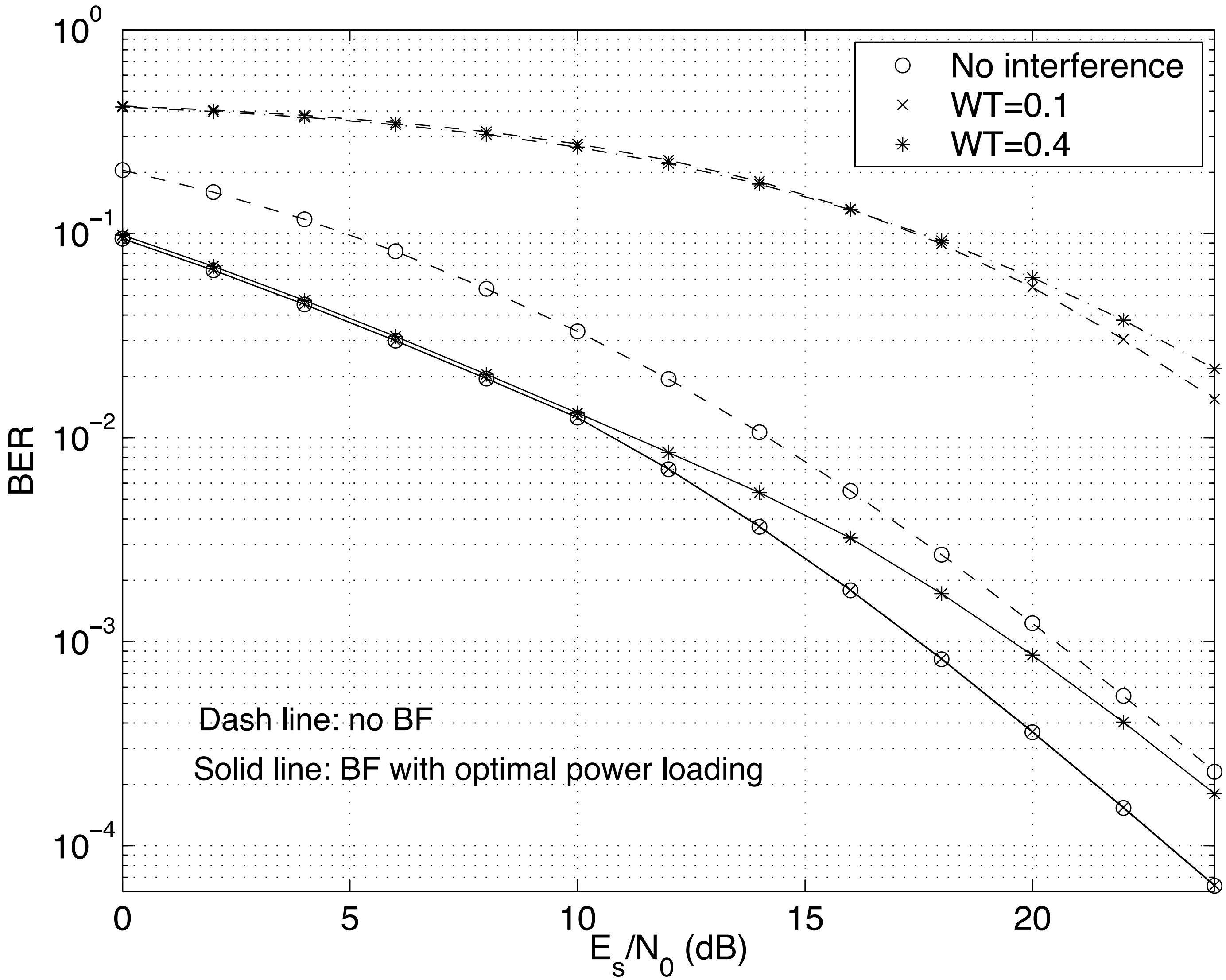
$$\text{subject to} \quad \sum_{i=1}^{N_{\min}} [\mathbf{D}]_{ii}^2 = 1.$$

Solution: $$[\mathbf{D}]_{ii}^2 = \left[\frac{1}{\bar{N}} + \frac{\mathcal{E}_s d_{\min}^2}{4N_0} \left(\frac{1}{\bar{N}} \sum_{j=1}^{\bar{N}} \frac{\lambda_{w,j}}{\lambda_{h,j}} - \frac{\lambda_{w,i}}{\lambda_{h,i}} \right) \right]_+,$$

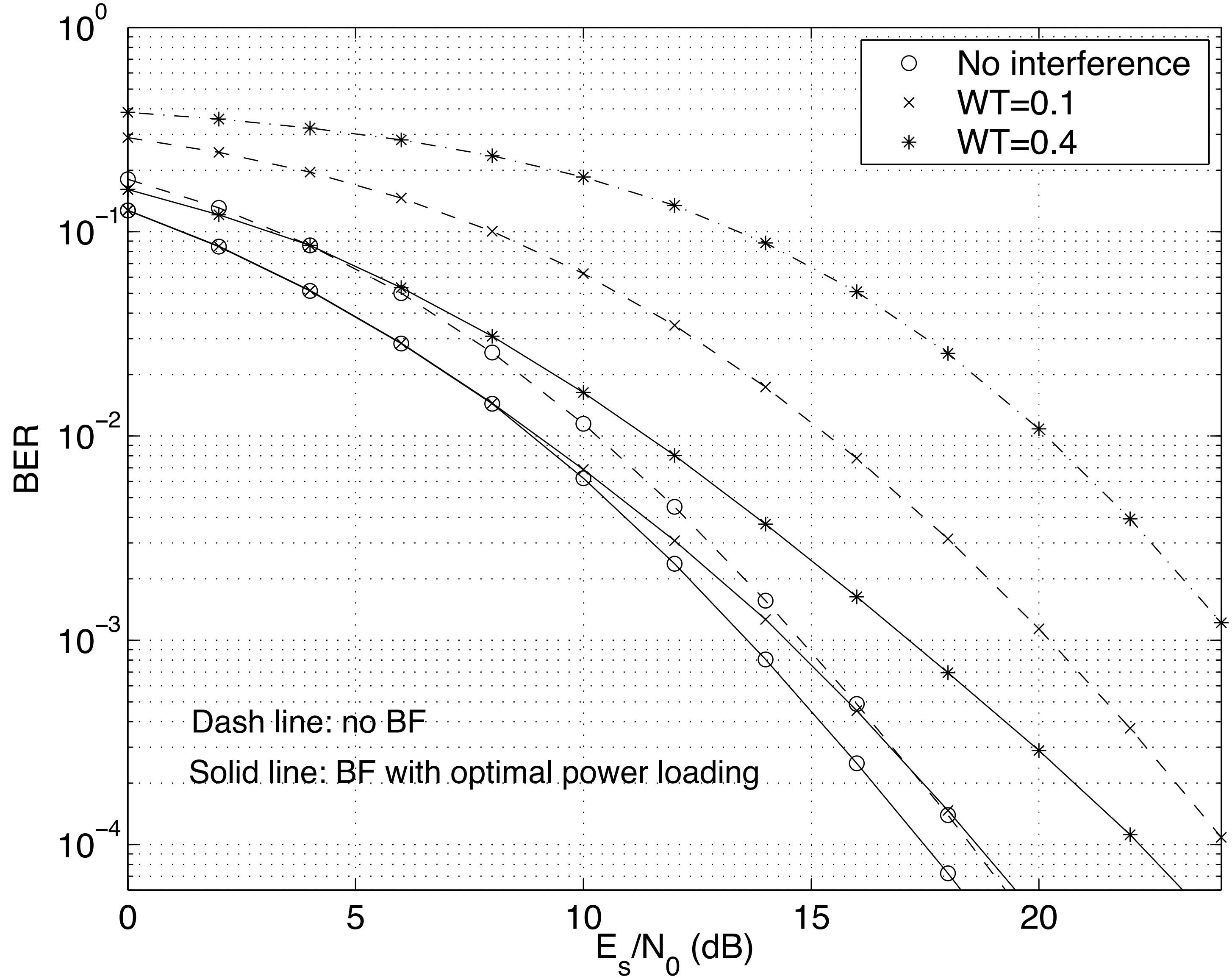
MISO Simulation Parameters

- $N_t = 4$ Tx antennas equi-spaced by d; Δ = multipath angular spread
- ring channel model: $[\mathbf{R}_h]_{m,n} \approx \frac{1}{2\pi}\int_0^{2\pi} \exp[-j2\pi(m-n)d\Delta\sin\theta/\lambda]d\theta$
- channel 1: $d = 0.5\lambda$ & $\Delta = 5°$; highly correlated with eigenvalues: $\mathbf{\Lambda}_{h_1} = \mathrm{diag}(3.81849, 0.18079, 0.00071, 0.00001)$
- channel 2: $d = 0.5\lambda$ & $\Delta = 25°$; less correlated with eigenvalues: $\mathbf{\Lambda}_{h_2} = \mathrm{diag}(1.790, 1.741, 0.454, 0.015)$
- simulated partial band interference with PSD $S(f) = \sin(\pi W f)/(\pi W f)$
- interference scenario 1: $W = 0.1/T \Rightarrow$ eigenvalues of $\mathbf{R}_i$: $\mathbf{\Lambda}_{i_1} = \mathrm{diag}(3.8412, 0.1579, 0.0009, 0.0000)$
- interference scenario 2: $W = 0.4/T \Rightarrow$ eigenvalues of $\mathbf{R}_i$: $\mathbf{\Lambda}_{i_2} = \mathrm{diag}(2.391, 1.394, 0.210, 0.005)$
- QPSK data/information symbols

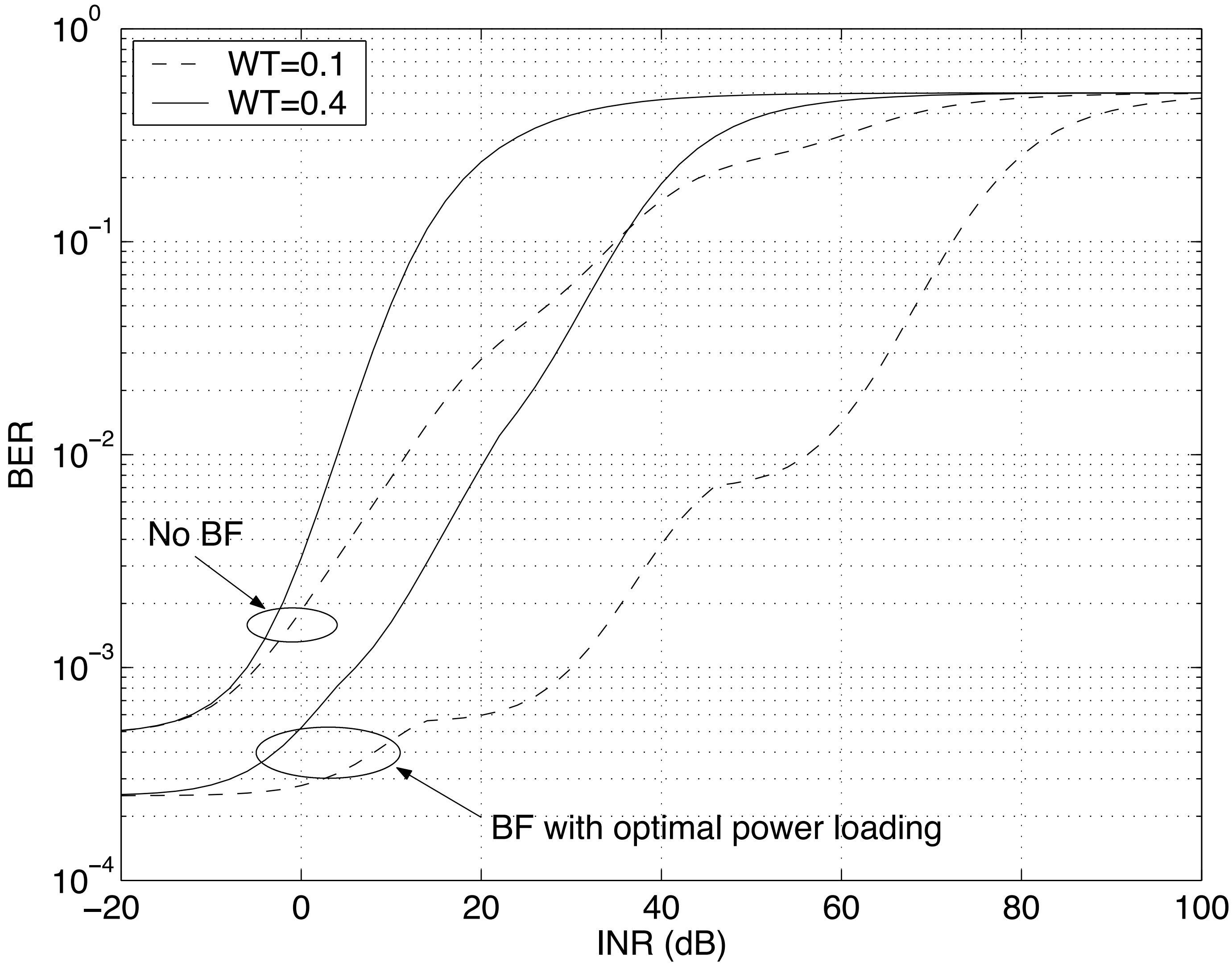
BER of STS transmissions, Channel 1, INR=10dB



BER of STS transmissions, Channel 2, INR=10dB



BER of STS transmissions versus INR, $\mathcal{E}_f/\mathcal{N}_0$=16dB, Channel 2.



MISO STBF/STBC w/ Interference Mitigation

- as an illustrative example, consider the rate 3/4 orthogonal STBC:

$$\mathbf{S} = \begin{pmatrix} s_3 & 0 & s_2 & s_1 \\ 0 & s_3 & s_1^* & -s_2^* \\ s_2^* & s_1 & -s_3^* & 0 \\ s_1^* & -s_2 & 0 & -s_3^* \end{pmatrix}$$

- an orthogonal space-time block codeword constructed from complex symbols $\{s_k = s_k^R + js_k^I\}_{k=1}^K$ may be represented as:

$$\mathbf{S} = \sum_{k=1}^{K} (\boldsymbol{\Phi}_k s_k^R + j\boldsymbol{\Psi}_k s_k^I),$$

- where matrices $\{\boldsymbol{\Phi}_k\}_{k=1}^K$ and $\{\boldsymbol{\Psi}_k\}_{k=1}^K$ have entries taken from $\{\pm 1, 0\}$
- without channel knowledge, each column of $\mathbf{S}$ is transmitted across the N_t antennas over N symbol periods

MISO STBF/STBC w/ Interference Mitigation

- with Tx knowledge of $\mathbf{R}_h$ and $\mathbf{R}_w$, STBF/STBC hybrid transmits signal block:

$$\mathbf{X} = \sqrt{\mathcal{E}_s}\mathbf{U}_w\mathbf{PSDU}_h^{\mathcal{H}}$$

- received block may be expressed as below, where $\mathbf{w}$ contains both interference and AWGN

$$\mathbf{y} = \mathbf{Xh} + \mathbf{w}$$

- ML receiver may be expressed as below, where $\tilde{\mathbf{y}} = \mathbf{U}_w^{\mathcal{H}}\mathbf{y}$

$$\hat{\mathbf{S}} = \arg_{\mathbf{S}} \min |\mathbf{\Lambda}_w^{-1/2}(\tilde{\mathbf{y}} - \sqrt{\mathcal{E}_s}\mathbf{PSDU}_h^{\mathcal{H}}\mathbf{h})|^2$$

- use exhaustive search or low-complexity search developed by Giannakis

- consider single error PEP $P_k := P(\mathbf{S} \to \tilde{\mathbf{S}})$ with $\mathbf{S} - \tilde{\mathbf{S}} = \mathbf{\Phi}_k e_k^R + j\mathbf{\Psi}_k e_k^I$ and $|e_k^R + je_k^I|^2 = d_{\min}^2$, since overall error probability is strongly affected by these signal error PEPs

$$P_{k,bound} = \left| \mathbf{I}_{N_t} + \frac{\mathcal{E}_s}{4N_0} \mathbf{D}(\mathbf{\Phi}_k e_k^R + j\mathbf{\Psi}_k e_k^I)^{\mathcal{H}} \mathbf{P}^{\mathcal{H}} \mathbf{\Lambda}_w^{-1} \mathbf{P}(\mathbf{\Phi}_k e_k^R + j\mathbf{\Psi}_k e_k^I) \mathbf{D} \mathbf{\Lambda}_h \right|^{-1}$$

- permutation matrix $\mathbf{P}$ and diagonal power loading matrix $\mathbf{D}$ chosen to minimize sum of $P_{k,bound}$'s

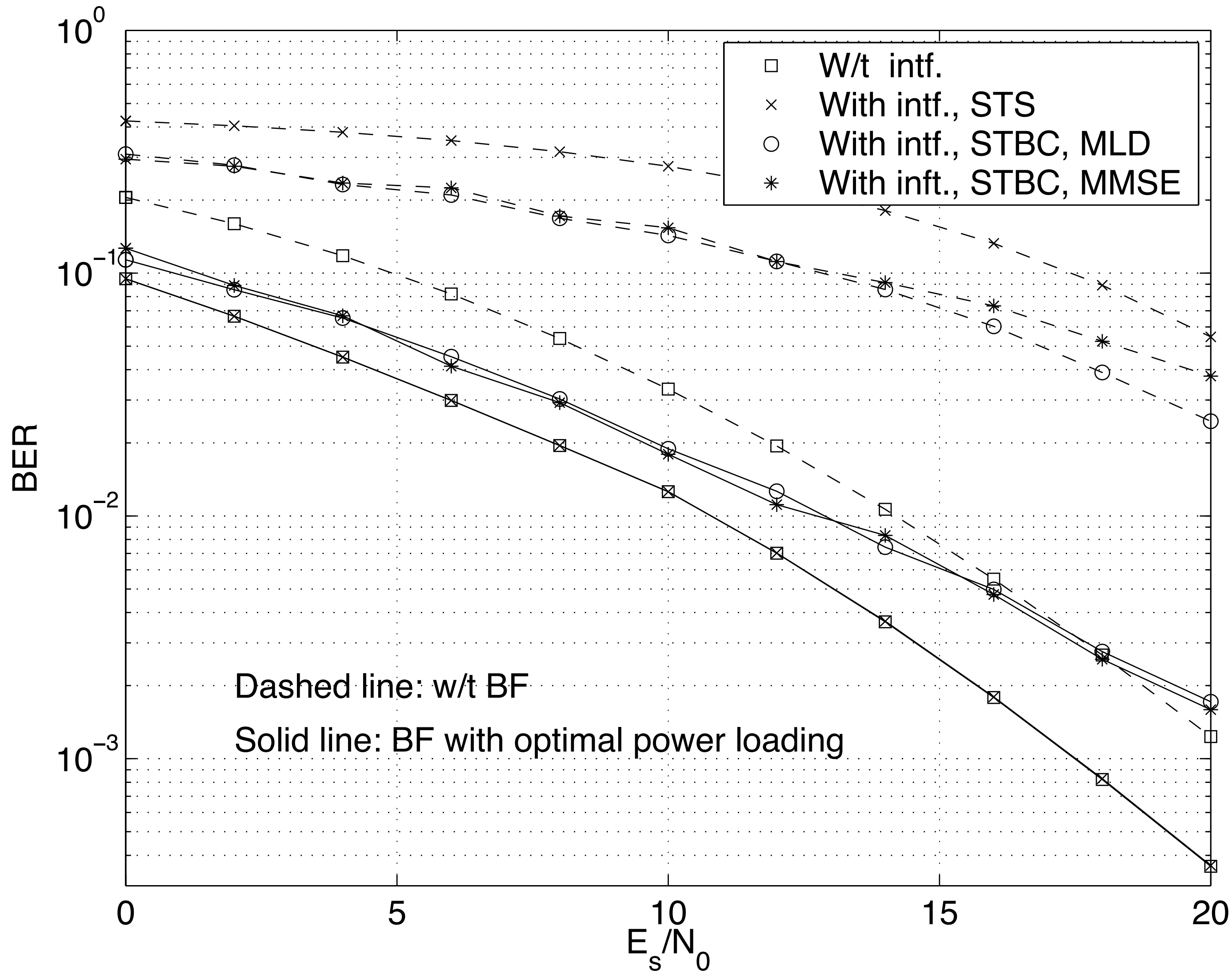
- Choose $\mathbf{P}$ so that large diagonal entries of $\mathbf{\Lambda}_h$ coincide with large diagonal entries of $\mathbf{Q}_k$, $\forall k$

- sequential quadratic programming may be used to solve below for the optimal $\mathbf{D}$

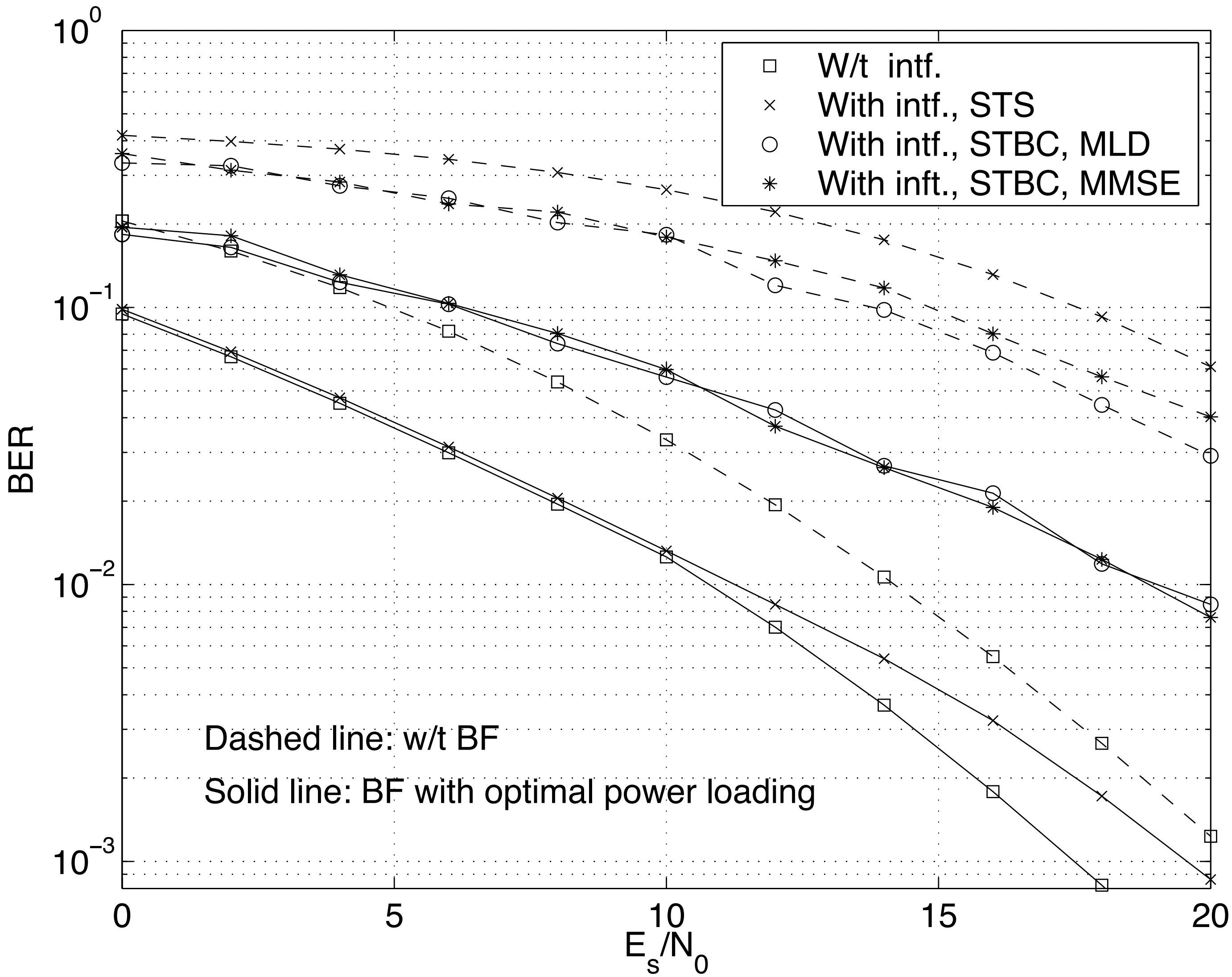
$$\mathbf{D}_{opt} = \arg_{\mathbf{D}} \min \sum_{k=1}^{K} P_{k,bound} \quad (10)$$

$$\text{subject to } \mathrm{Tr}(\mathbf{D}^2) = 1$$

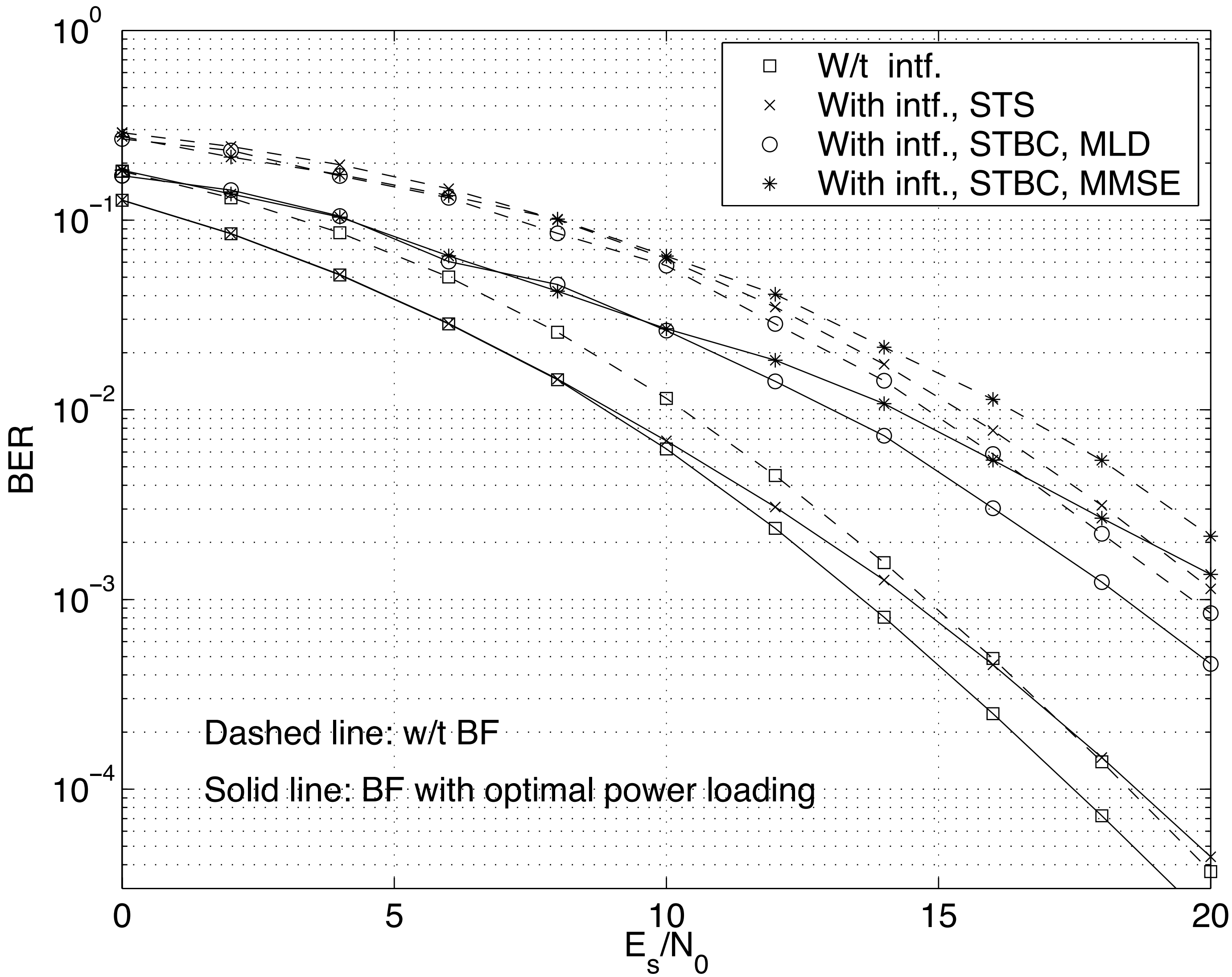
Simulated BER of STBC, INR=10dB, Channel 1, $WT = 0.1$.



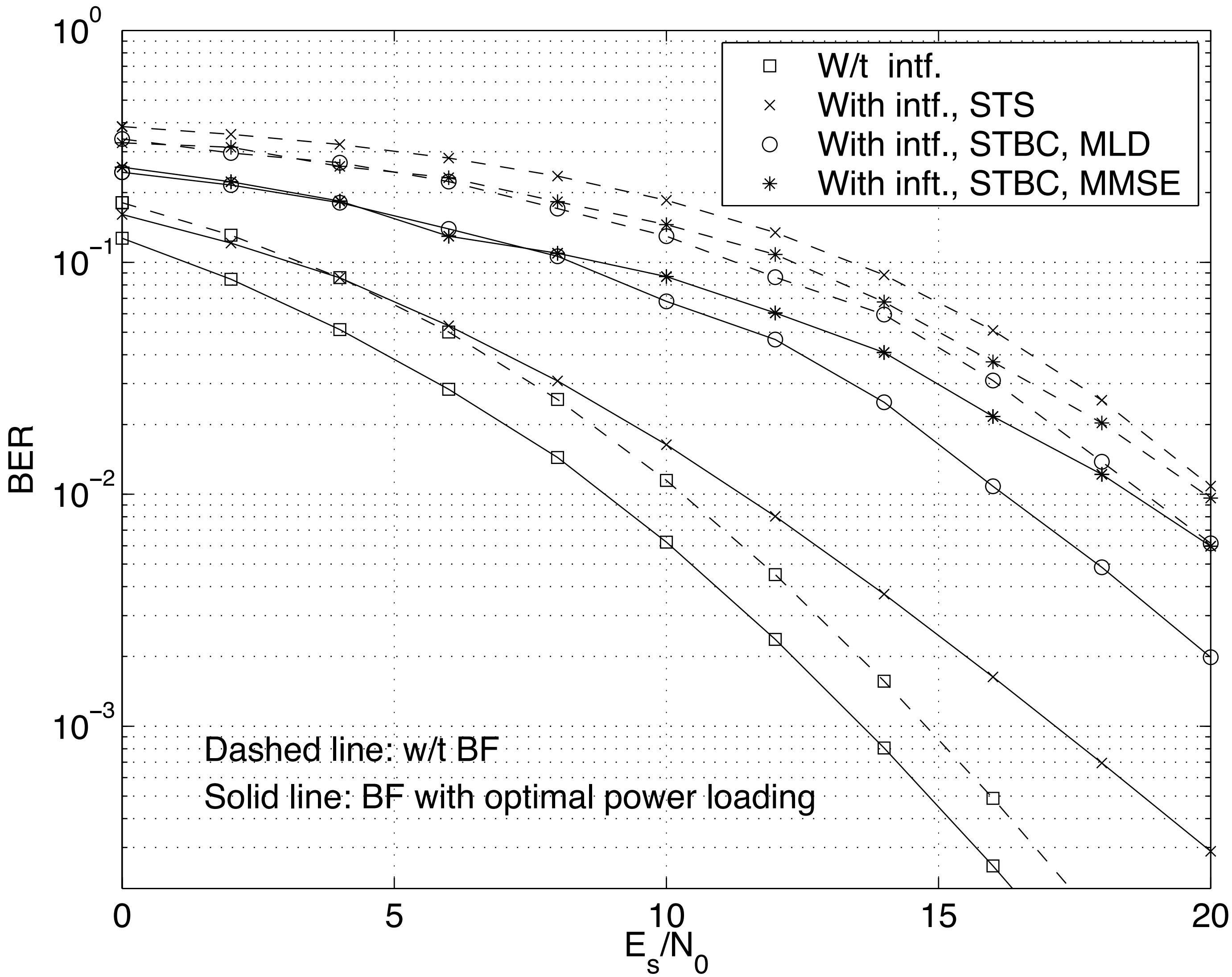
Simulated BER of STBC, INR=10dB, Channel 1, $WT = 0.4$.



Simulated BER of STBC, INR=10dB, Channel 2, $WT = 0.1$.



Simulated BER of STBC, INR=10dB, Channel 2, $WT = 0.4$.



- channel estimator for Rx: transmit training matrix $\mathbf{C}$ w/o info. symbol $\Rightarrow \mathbf{y} = \mathbf{C}\mathbf{h} + \mathbf{w}$. LSE channel estimate:

$$\hat{\mathbf{h}} = (\mathbf{C}^{\mathcal{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathcal{H}}\mathbf{y} = \mathbf{h} + (\mathbf{C}^{\mathcal{H}}\mathbf{C})^{-1}\mathbf{C}^{\mathcal{H}}\mathbf{w}$$

- MMSE of LSE channel estimator achieved with $\mathbf{C} = \mathbf{U}_w\mathbf{D}\mathbf{V}_c^{\mathcal{H}}$, where $\mathbf{V}_c$ is arbitrary unitary matrix and $\mathbf{D}$ is diagonal with:

$$[\mathbf{D}]_{ii}^2 = \frac{\sqrt{\lambda_{w,i}}}{\sum_{j=1}^{N_t}\sqrt{\lambda_{w,j}}}$$

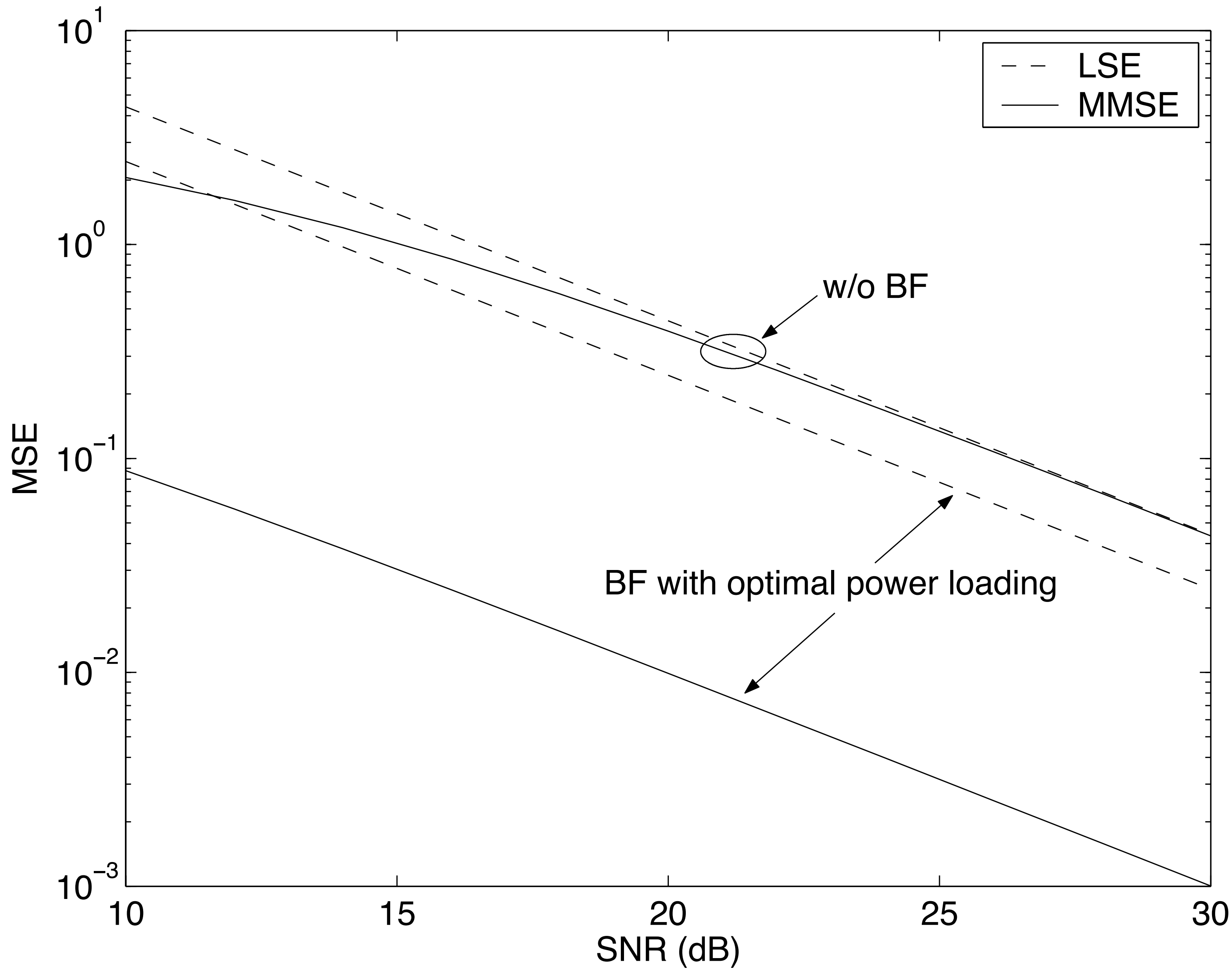
- linear MMSE channel estimate using knowledge of $\mathbf{R}_h$ as well as $\mathbf{R}_w$:

$$\hat{\mathbf{h}} = \sqrt{N\mathcal{E}_p}\mathbf{R}_h\mathbf{C}^{\mathcal{H}}(N\mathcal{E}_p\mathbf{C}\mathbf{R}_h\mathbf{C} + N_0\mathbf{R}_w)^{-1}\mathbf{y}$$

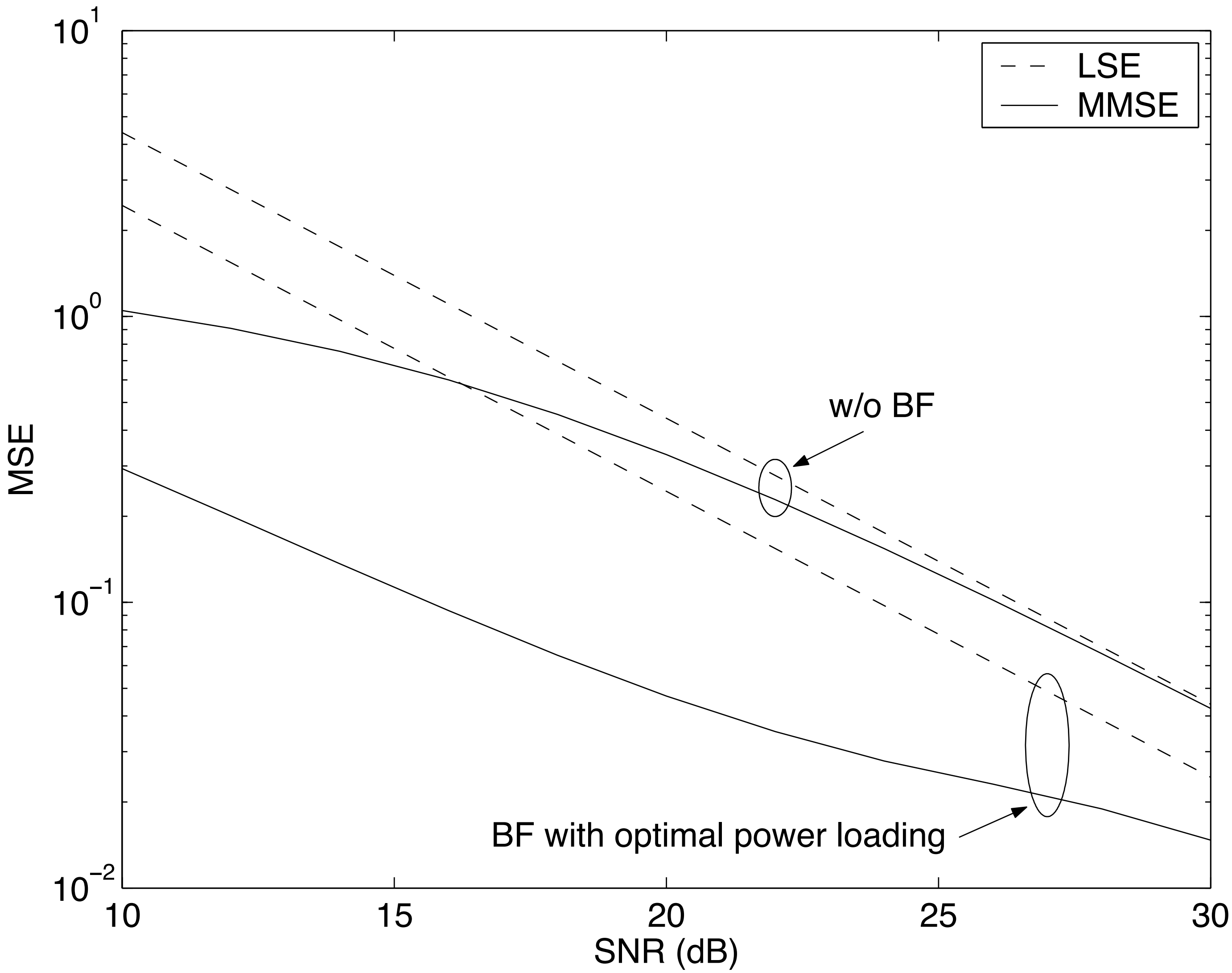
- MMSE of "MMSE" channel estimator achieved with $\mathbf{C} = \mathbf{U}_w\mathbf{D}\mathbf{V}_h^{\mathcal{H}}$, where $\mathbf{D}$ is diagonal with:

$$[\mathbf{D}]_{ii}^2 = \left[\frac{1 + \frac{N_0}{N\mathcal{E}_p}\sum_{j\in\mathcal{I}}\frac{\lambda_{w,j}}{\lambda_{h,j}}}{\sum_{j\in\mathcal{I}}\sqrt{\lambda_{w,j}}}\sqrt{\lambda_{w,i}} - \frac{N_0}{N\mathcal{E}_p}\frac{\lambda_{w,i}}{\lambda_{h,i}}\right]_+$$

Mean square error of channel estimators, INR=10dB, Channel 1, $WT = 0.1$.



Mean square error of channel estimators, INR=10dB, Channel 2, $WT = 0.1$.



STBF WITH INTERFERENCE: SINGLE RECEIVE ANTENNA

- exploited second-order spatial stats of channel & temporal statistics of interference to design transceivers for multi-antenna wireless communication systems
- space-time spreading via transmitting signals along strongest eigen-direction of channel and weakest eigen-direction of interference maximizes average SINR
- derived optimally power loaded space-time beam-forming (STBF) schemes – if strong channels coincide with weak interference, error probability reduces substantially
- to increase transmission rates, combined STBC with STBF, derive dpower loading schemes & low-complexity receivers
- investigated optimal training for LSE channel estimation and STBF for MMSE channel estimation
- STBF with optimal power loading substantially reduces error probability & channel estimator errors

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- SISO Temporal Processing with interference pre-cancelling and "STBC" option
- MISO Space-Time Beamforming with interference pre-cancelling and STBC option
- **MIMO Space-Only Beamforming with interference pre-cancelling & STBC option**
  - next development considers more general case of N Tx antennae and M Rx antennae but space-only processing at both ends
  - spatial correlation matrix of inteference fed back from Rx to Tx along with channel gains
- MIMO Space-Time Beamforming with interference pre-cancelling and STBC option

STBF/STBC Hybrid with Interference Mitigation: Spatial Processing

- PROBLEM ADDRESSED: cross-talk amongst eigen-beams at receiver with transmit eigen-beamforming based on imperfect channel knowledge
- SOLUTION: STBF/STBC hybrid providing robustness to mismatch between true channel matrix and channel matrix estimate employed at transmitter to compute transmit beamforming weights
- STBF/STBC hybrid facilitates sharing of common eigen-beams (over time) averting need for power loading
- STBF/STBC hybrid incorporates pre-interference cancellation based on imperfect knowledge of (spatial) interference correlation matrix at receiver (obtained thru feedback similar to channel)

STBF/STBC Hybrid with Interference Mitigation: Spatial Processing

- Let $\boldsymbol{\mathcal{H}}$ denote the $M \times N$ channel matrix, with $\boldsymbol{\mathcal{H}}_{ij}$ equal to the channel gain from the j-th transmit antenna, $i = 1, ..., N$, to the i-th receive antenna, $j = 1, ..., M$ $\quad$ (N Tx antennas - M Rx antennas)
- Consider space-only beamforming at both transmitter and receiver
- at time n, we use $N \times 1$ beamforming weight vector $\mathbf{w}_k$ to transmit symbol $s[n]$
- The $M \times 1$ signal vector at the receiver is then $\boldsymbol{\mathcal{H}}\mathbf{w}_k$ such that the (spatial) white noise matched filter is $\mathbf{w}_{R_k} = \boldsymbol{\mathcal{H}}\mathbf{w}_k$
- next consider multiple symbols transmitted simultaneously using distinct transmit beamforming weight vectors
- For simplicity and ease of illustration, consider case of only two transmit beams

STBF/STBC Hybrid w/ Interference Mitigation: Spatial Processing

- consider Alamouti space-time coding at the beam level according to:

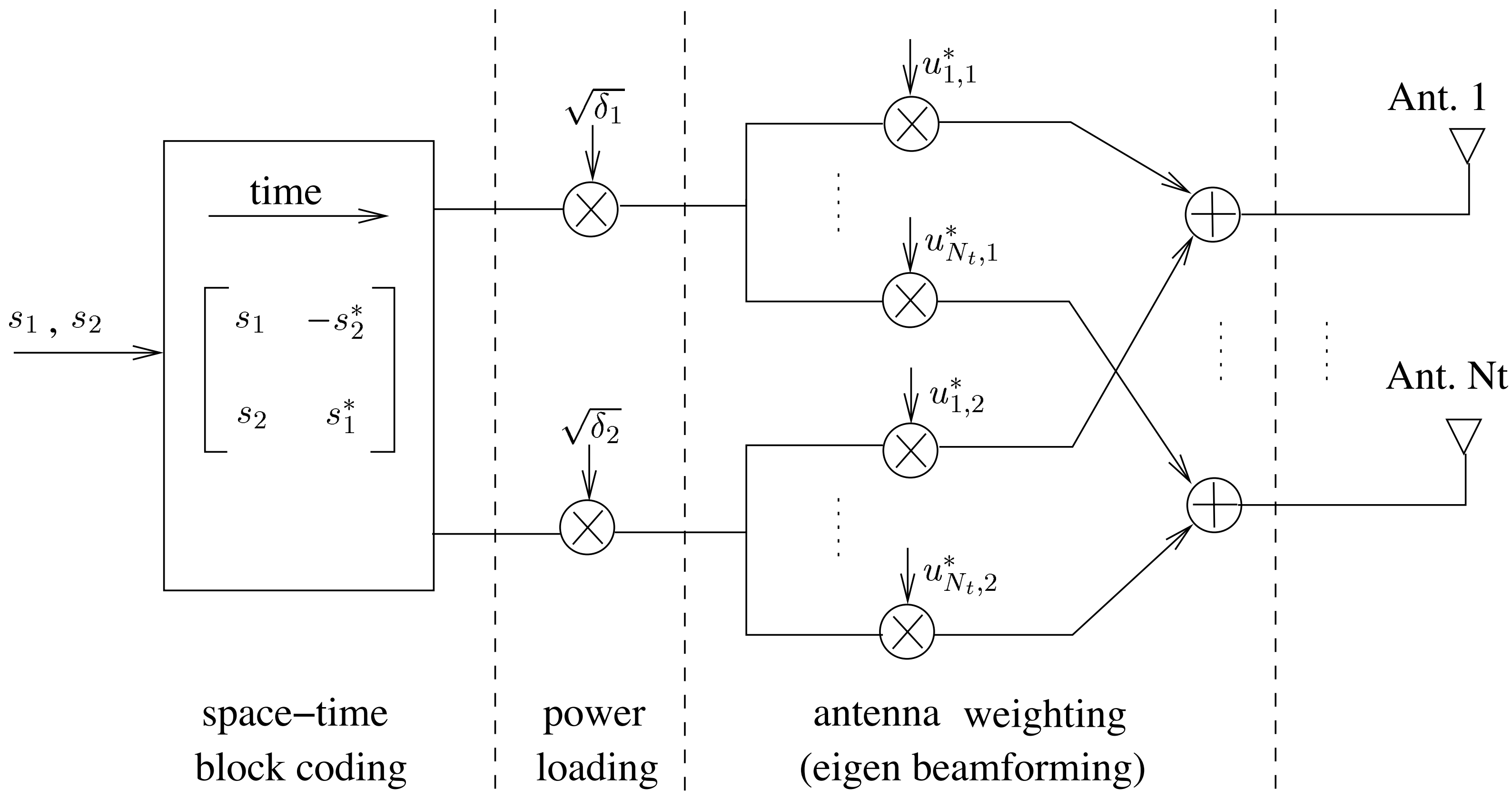
$$\begin{aligned}\mathbf{x}[n] &= s[n]\mathbf{w}_k + s[n{+}1]\mathbf{w}_\ell \\ \mathbf{x}[n{+}1] &= s^*[n{+}1]\mathbf{w}_k - s^*[n]\mathbf{w}_\ell\end{aligned}$$

- note: transmit beamforming weight vectors are not conjugated at time n+1 relative to time n, even though the symbols are conjugated
- Let $\mathbf{y}[n]$ & $\mathbf{y}$[n+1] denote $M \times 1$ received signal vectors at time n and n+1, respectively:

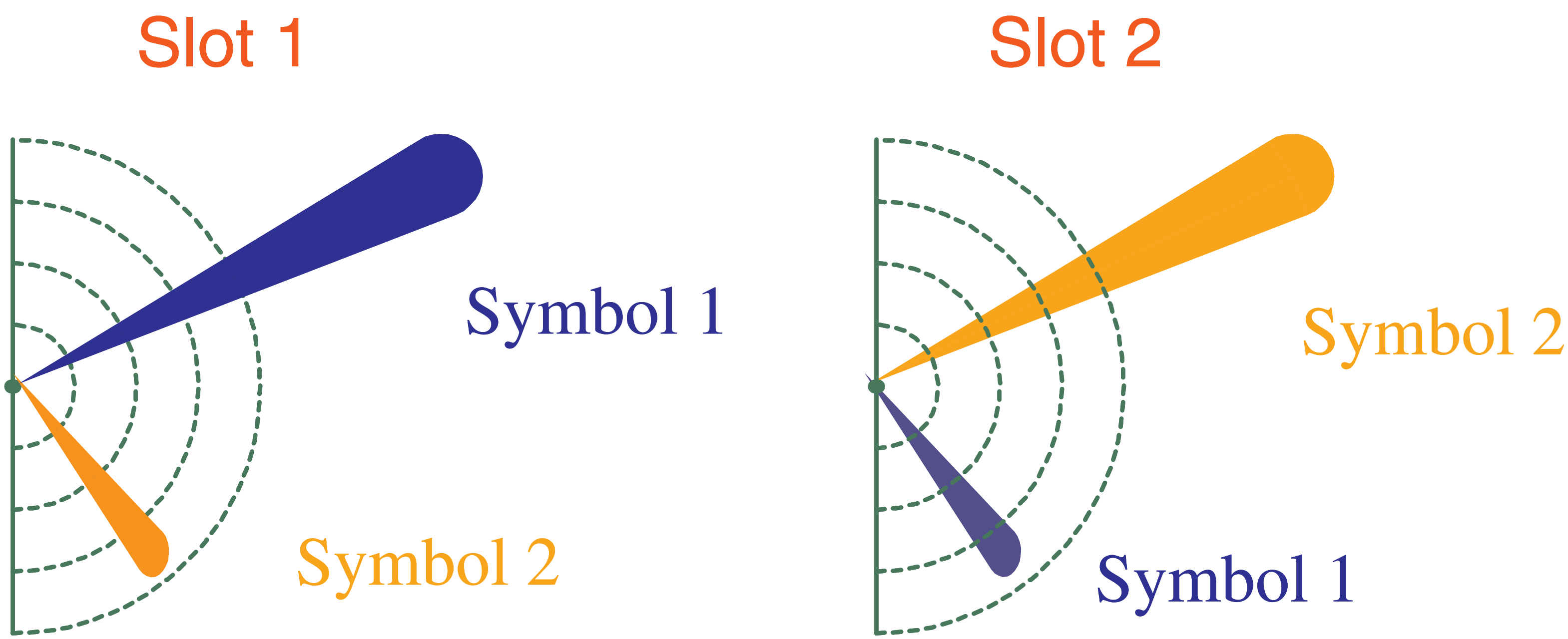
$$\begin{aligned}\mathbf{y}[n] &= s[n]\boldsymbol{\mathcal{H}}\mathbf{w}_k + s[n{+}1]\boldsymbol{\mathcal{H}}\mathbf{w}_\ell + \boldsymbol{\nu}[n] \\ \mathbf{y}^*[n{+}1] &= s[n{+}1]\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^* - s[n]\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^* + \boldsymbol{\nu}^*[n{+}1]\end{aligned}$$

- noise is assumed both temporally and spatially white

Simple Example of STBF/STBC Hybrid



Simple Example of STBF/STBC Hybrid



STBF/STBC Hybrid w/ Interference Mitigation: Spatial Processing

- applying spatial matched filters, $\boldsymbol{\mathcal{H}}\mathbf{w}_k$ and $\boldsymbol{\mathcal{H}}\mathbf{w}_\ell$, at time n to $\mathbf{y}[n]$:

$$\begin{aligned} z_k[n] &= \mathbf{w}_k^H \boldsymbol{\mathcal{H}}^H \mathbf{y}[n] \\ &= s[n]\mathbf{w}_k^H \boldsymbol{\mathcal{H}}^H \boldsymbol{\mathcal{H}} \mathbf{w}_k + s[n+1]\mathbf{w}_k^H \boldsymbol{\mathcal{H}}^H \boldsymbol{\mathcal{H}} \mathbf{w}_\ell + \mathbf{w}_k^H \boldsymbol{\mathcal{H}}^H \boldsymbol{\nu}[n] \\ z_\ell[n] &= \mathbf{w}_\ell^H \boldsymbol{\mathcal{H}}^H \mathbf{y}[n] \\ &= s[n]\mathbf{w}_\ell^H \boldsymbol{\mathcal{H}}^H \boldsymbol{\mathcal{H}} \mathbf{w}_k + s[n+1]\mathbf{w}_\ell^H \boldsymbol{\mathcal{H}}^H \boldsymbol{\mathcal{H}} \mathbf{w}_\ell + \mathbf{w}_\ell^H \boldsymbol{\mathcal{H}}^H \boldsymbol{\nu}[n] \end{aligned}$$

- applying spatial matched filters, $\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^*$ and $\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^*$, at time n+1 to $\mathbf{y}^*$[n+1]:

$$\begin{aligned} z_k[n+1] &= \mathbf{w}_k^T \boldsymbol{\mathcal{H}}^T \mathbf{y}^*[n+1] \\ &= s[n+1]\mathbf{w}_k^T \boldsymbol{\mathcal{H}}^T \boldsymbol{\mathcal{H}}^* \mathbf{w}_k^* - s[n]\mathbf{w}_k^T \boldsymbol{\mathcal{H}}^T \boldsymbol{\mathcal{H}}^* \mathbf{w}_\ell^* + \mathbf{w}_k^T \boldsymbol{\mathcal{H}}^T \boldsymbol{\nu}^*[n+1] \\ z_\ell[n+1] &= \mathbf{w}_\ell^T \boldsymbol{\mathcal{H}}^T \mathbf{y}^*[n+1] \\ &= s[n+1]\mathbf{w}_\ell^T \boldsymbol{\mathcal{H}}^T \boldsymbol{\mathcal{H}}^* \mathbf{w}_k^* - s[n]\mathbf{w}_\ell^T \boldsymbol{\mathcal{H}}^T \boldsymbol{\mathcal{H}}^* \mathbf{w}_\ell^* + \mathbf{w}_\ell^T \boldsymbol{\mathcal{H}}^T \boldsymbol{\nu}^*[n+1] \end{aligned}$$

STBF/STBC Hybrid w/ Interference Mitigation: Spatial Processing

- WLOG constrain Tx beamforming weights to have unit energy: $\mathbf{w}_k^H \mathbf{w}_k = 1$ & $\mathbf{w}_\ell^H \mathbf{w}_\ell = 1$
- for Rx noise to be uncorrelated in beamspace, require Rx beamforming vectors $\boldsymbol{\mathcal{H}}\mathbf{w}_k$ & $\boldsymbol{\mathcal{H}}\mathbf{w}_\ell$ to be orthogonal: $\mathbf{w}_k^H \boldsymbol{\mathcal{H}}^H \boldsymbol{\mathcal{H}} \mathbf{w}_\ell = 0$
- SNR at receiver at output of k-th spatial matched filter is then: $\propto \mathbf{w}_k^H \boldsymbol{\mathcal{H}}^H \boldsymbol{\mathcal{H}} \mathbf{w}_k$
- motivates choosing Tx beamforming weight vectors equal to “largest” eigenvectors of $\boldsymbol{\mathcal{H}}^H \boldsymbol{\mathcal{H}}$
- development generalizes for combining orthogonal space-time block codes with larger number of beams, but desire to reserve DOF’s for pre-interference mitigation (will incorporate shortly)
- *note:* numerous researchers, e.g., Eko O., have done excellent work on transmitting information thru eigen-channels

- collecting spatial matched filter outputs to $\boldsymbol{\mathcal{H}}\mathbf{w}_k$ and $\boldsymbol{\mathcal{H}}\mathbf{w}_\ell$ at time n with spatial matched filter outputs to $\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^*$ and $\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^*$ at time n+1:

$$\mathbf{z}[n] = s[n]\mathbf{b}_\mathrm{n} + s[n{+}1]\mathbf{b}_{n+1} + \boldsymbol{\nu}_B[n] \qquad (4 \times 1)$$

$$\mathbf{b}_\mathrm{n} = \begin{bmatrix} \mathbf{w}_k^H\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}}\mathbf{w}_k \\ \mathbf{w}_\ell^H\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}}\mathbf{w}_k \\ -\mathbf{w}_k^T\boldsymbol{\mathcal{H}}^T\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^* \\ -\mathbf{w}_\ell^T\boldsymbol{\mathcal{H}}^T\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^* \end{bmatrix} \qquad \mathbf{b}_{n+1} = \begin{bmatrix} \mathbf{w}_k^H\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}}\mathbf{w}_\ell \\ \mathbf{w}_\ell^H\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}}\mathbf{w}_\ell \\ \mathbf{w}_k^T\boldsymbol{\mathcal{H}}^T\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^* \\ \mathbf{w}_\ell^T\boldsymbol{\mathcal{H}}^T\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^* \end{bmatrix}$$

- employing eigen-beams with perfect channel knowledge:

$$\mathbf{b}_\mathrm{n} = \begin{bmatrix} \lambda_k \\ 0 \\ 0 \\ -\lambda_\ell \end{bmatrix} \qquad \mathbf{b}_{n+1} = \begin{bmatrix} 0 \\ \lambda_\ell \\ \lambda_k \\ 0 \end{bmatrix}$$

- $\mathbf{b}_\mathrm{n}^H\mathbf{z}[n]$ and $\mathbf{b}_{n+1}^H\mathbf{z}[n]$ yield same SNR gain per symbol: $\lambda_k^2 + \lambda_\ell^2$

- *important:* STBF/STBC hybrid provides robustness to mismatch between true channel matrix, $\boldsymbol{\mathcal{H}}$, and estimated channel matrix, $\hat{\boldsymbol{\mathcal{H}}}$, used to compute Tx beamforming weight vectors $\hat{\mathbf{w}}_k$ and $\hat{\mathbf{w}}_\ell$
- with mismatch due to estimation error, quantization, and/or mobility:

$$\mathbf{b}_{\mathrm{n}} = \begin{bmatrix} \hat{\mathbf{w}}_k^H \hat{\boldsymbol{\mathcal{H}}}^H \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_k \\ \hat{\mathbf{w}}_\ell^H \hat{\boldsymbol{\mathcal{H}}}^H \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_k \\ -\hat{\mathbf{w}}_k^T \hat{\boldsymbol{\mathcal{H}}}^T \boldsymbol{\mathcal{H}}^* \hat{\mathbf{w}}_\ell^* \\ -\hat{\mathbf{w}}_\ell^T \hat{\boldsymbol{\mathcal{H}}}^T \boldsymbol{\mathcal{H}}^* \hat{\mathbf{w}}_\ell^* \end{bmatrix} \qquad \mathbf{b}_{n+1} = \begin{bmatrix} \hat{\mathbf{w}}_k^H \hat{\boldsymbol{\mathcal{H}}}^H \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_\ell \\ \hat{\mathbf{w}}_\ell^H \hat{\boldsymbol{\mathcal{H}}}^H \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_\ell \\ \hat{\mathbf{w}}_k^T \hat{\boldsymbol{\mathcal{H}}}^T \boldsymbol{\mathcal{H}}^* \hat{\mathbf{w}}_k^* \\ \hat{\mathbf{w}}_\ell^T \hat{\boldsymbol{\mathcal{H}}}^T \boldsymbol{\mathcal{H}}^* \hat{\mathbf{w}}_k^* \end{bmatrix}$$

- cross-terms like $\hat{\mathbf{w}}_k^H \hat{\boldsymbol{\mathcal{H}}}^H \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_\ell$ are no longer zero, and “auto-terms” like $\hat{\mathbf{w}}_k^H \hat{\boldsymbol{\mathcal{H}}}^H \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_k$ are no longer real-valued
- $\mathbf{b}_{\mathrm{n}}$ and $\mathbf{b}_{\mathrm{n}+1}$ still orthogonal AND yield same SNR gain per symbol!!
- despite channel feedback, Rx must estimate 4 values comprising $\mathbf{b}_{\mathrm{n}}$

STBF/STBC Hybrid w/ Interference Mitigation: Spatial Processing

- consider Alamouti space-time coding at the beam level according to:

$$\begin{aligned}\mathbf{x}[n] &= s[n]\mathbf{w}_k + s[n{+}1]\mathbf{w}_\ell \\ \mathbf{x}[n{+}1] &= s^*[n{+}1]\mathbf{w}_k - s^*[n]\mathbf{w}_\ell\end{aligned}$$

- note: transmit beamforming weight vectors are not conjugated at time n+1 relative to time n, even though the symbols are conjugated
- Let $\mathbf{y}[n]$ & $\mathbf{y}$[n+1] denote $M \times 1$ received signal vectors at time n and n+1, respectively:

$$\begin{aligned}\mathbf{y}[n] &= s[n]\boldsymbol{\mathcal{H}}\mathbf{w}_k + s[n{+}1]\boldsymbol{\mathcal{H}}\mathbf{w}_\ell + \boldsymbol{\nu}_{I+N}[n] \\ \mathbf{y}^*[n{+}1] &= s[n{+}1]\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^* - s[n]\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^* + \boldsymbol{\nu}_{I+N}^*[n{+}1]\end{aligned}$$

- $\boldsymbol{\nu}_{I+N}[n]$: represents interference plus noise contribution with spatial covariance matrix $\mathbf{R}_{I+N}$

- applying optimum spatial MMSE filters, $\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_k$ and $\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_\ell$, at time n to $\mathbf{y}[n]$:

$$
\begin{aligned}
z_k[n] &= \mathbf{w}_k^H\boldsymbol{\mathcal{H}}^H\mathbf{R}_{I+N}^{-1}\mathbf{y}[n] \\
&= s[n]\mathbf{w}_k^H\boldsymbol{\mathcal{H}}^H\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_k + s[n{+}1]\mathbf{w}_k^H\boldsymbol{\mathcal{H}}^H\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_\ell + \mathbf{w}_k^H\boldsymbol{\mathcal{H}}^H\mathbf{R}_{I+N}^{-1}\boldsymbol{\nu}_{I+N}[\\
z_\ell[n] &= \mathbf{w}_\ell^H\boldsymbol{\mathcal{H}}^H\mathbf{y}[n] \\
&= s[n]\mathbf{w}_\ell^H\boldsymbol{\mathcal{H}}^H\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_k + s[n{+}1]\mathbf{w}_\ell^H\boldsymbol{\mathcal{H}}^H\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_\ell + \mathbf{w}_\ell^H\boldsymbol{\mathcal{H}}^H\mathbf{R}_{I+N}^{-1}\boldsymbol{\nu}_{I+N}[
\end{aligned}
$$

- applying spatial matched filters, $\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^*$ and $\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^*$, at time n+1 to $\mathbf{y}^*$[n+1]:

$$
\begin{aligned}
z_k[n{+}1] &= \mathbf{w}_k^T\boldsymbol{\mathcal{H}}^T\mathbf{R}_{I+N}^{-1*}\mathbf{y}^*[n{+}1] \\
&= s[n{+}1]\mathbf{w}_k^T\boldsymbol{\mathcal{H}}^T\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^* - s[n]\mathbf{w}_k^T\boldsymbol{\mathcal{H}}^T\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^* + \mathbf{w}_k^T\boldsymbol{\mathcal{H}}^T\mathbf{R}_{I+N}^{-1*}\boldsymbol{\nu}_{I-}^* \\
z_\ell[n{+}1] &= \mathbf{w}_\ell^T\boldsymbol{\mathcal{H}}^T\mathbf{R}_{I+N}^{-1*}\mathbf{y}^*[n{+}1] \\
&= s[n{+}1]\mathbf{w}_\ell^T\boldsymbol{\mathcal{H}}^T\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^* - s[n]\mathbf{w}_\ell^T\boldsymbol{\mathcal{H}}^T\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^* + \mathbf{w}_\ell^T\boldsymbol{\mathcal{H}}^T\mathbf{R}_{I+N}^{-1*}\boldsymbol{\nu}_{I-}^*
\end{aligned}
$$

STBF/STBC Hybrid w/ Interference Mitigation: Spatial Processing

- WLOG constrain Tx beamforming weights to have unit energy: $\mathbf{w}_k^H \mathbf{w}_k = 1$ & $\mathbf{w}_\ell^H \mathbf{w}_\ell = 1$
- for Rx interference plus noise to be uncorrelated in beamspace, require Rx MMSE beamforming vectors $\mathbf{v}_k = \mathbf{R}_{I+N}^{-1} \boldsymbol{\mathcal{H}} \mathbf{w}_k$ & $\mathbf{v}_\ell = \mathbf{R}_{I+N}^{-1} \boldsymbol{\mathcal{H}} \mathbf{w}_\ell$ to be "orthogonal" in Hilbert space with inner product $\mathbf{v}_k^H \mathbf{R}_{I+N} \mathbf{v}_\ell \Rightarrow$ $\mathbf{w}_k^H \boldsymbol{\mathcal{H}}^H \mathbf{R}_{I+N}^{-1} \boldsymbol{\mathcal{H}} \mathbf{w}_\ell = 0$
- SINR at receiver at output of k-th spatial matched filter is then: $\propto \mathbf{w}_k^H \boldsymbol{\mathcal{H}}^H \mathbf{R}_{I+N}^{-1} \boldsymbol{\mathcal{H}} \mathbf{w}_k$
- motivates choosing Tx beamforming weight vectors equal to "largest" eigenvectors of $\boldsymbol{\mathcal{H}}^H \mathbf{R}_{I+N}^{-1} \boldsymbol{\mathcal{H}}$

STBF/STBC Hybrid w/ Interference Mitigation: Spatial Processing

- Recall: $\boldsymbol{\mathcal{H}}_{ij}$ is channel gain from j-th Tx antenna to i-th Rx antenna – N transmit antennas, M receive antennas

$$\boldsymbol{\mathcal{H}} = \left[\, \mathbf{h}_1 \vdots \mathbf{h}_2 \vdots \ldots \vdots \mathbf{h}_N \, \right]$$

- $\mathbf{h}_j$: $M \times 1$ vector of channel gains from j-th transmit antenna to all M receive antennas
- Tx beamforming wt. vectors: "largest" eigenvectors of $\boldsymbol{\mathcal{H}}^H \mathbf{R}_{I+N}^{-1} \boldsymbol{\mathcal{H}}$

$$\mathcal{E}\left\{\boldsymbol{\mathcal{H}}^H \mathbf{R}_{I+N}^{-1} \boldsymbol{\mathcal{H}}\right\} = \sum_{k=1}^{M} \sum_{\ell=1}^{M} \mathbf{R}_{I+N_{k,\ell}}^{-1} \mathcal{E}\{\mathbf{h}_k \mathbf{h}_\ell^H\}$$

- collecting spatial MMSE filter outputs to $\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_k$ and $\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_\ell$ at time n with spatial MMSE filter outputs to $\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^*$ and $\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^*$ at time n+1:

$$\mathbf{z}[n] = s[n]\mathbf{b}_{\mathrm{n}} + s[n{+}1]\mathbf{b}_{n+1} + \boldsymbol{\nu}_B[n] \qquad (4\times 1)$$

$$\mathbf{b}_{\mathrm{n}} = \begin{bmatrix} \mathbf{w}_k^H\boldsymbol{\mathcal{H}}^H\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_k \\ \mathbf{w}_\ell^H\boldsymbol{\mathcal{H}}^H\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_k \\ -\mathbf{w}_k^T\boldsymbol{\mathcal{H}}^T\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^* \\ -\mathbf{w}_\ell^T\boldsymbol{\mathcal{H}}^T\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_\ell^* \end{bmatrix} \qquad \mathbf{b}_{n+1} = \begin{bmatrix} \mathbf{w}_k^H\boldsymbol{\mathcal{H}}^H\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_\ell \\ \mathbf{w}_\ell^H\hat{\boldsymbol{\mathcal{H}}}^H\mathbf{R}_{I+N}^{-1}\boldsymbol{\mathcal{H}}\mathbf{w}_\ell \\ \mathbf{w}_k^T\hat{\boldsymbol{\mathcal{H}}}^T\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^* \\ \mathbf{w}_\ell^T\hat{\boldsymbol{\mathcal{H}}}^T\mathbf{R}_{I+N}^{-1*}\boldsymbol{\mathcal{H}}^*\mathbf{w}_k^* \end{bmatrix}$$

- using eigen-beams with perfect channel & interference knowledge:

$$\mathbf{b}_{\mathrm{n}} = \begin{bmatrix} \lambda_k \\ 0 \\ 0 \\ -\lambda_\ell \end{bmatrix} \qquad \mathbf{b}_{n+1} = \begin{bmatrix} 0 \\ \lambda_\ell \\ \lambda_k \\ 0 \end{bmatrix}$$

- *important:* STBF/STBC hybrid provides robustness to mismatch between true channel matrix, $\boldsymbol{\mathcal{H}}$, and estimated channel matrix, $\hat{\boldsymbol{\mathcal{H}}}$, AND to mismatch between true $\mathbf{R}_{I+N}$ and estimated $\hat{\mathbf{R}}_{I+N}^{-1}$, used to compute Tx beamforming weight vectors $\hat{\mathbf{w}}_k$ and $\hat{\mathbf{w}}_\ell$

$$\mathbf{b}_{\mathrm{n}} = \begin{bmatrix} \hat{\mathbf{w}}_k^H \hat{\boldsymbol{\mathcal{H}}}^H \hat{\mathbf{R}}_{I+N}^{-1} \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_k \\ \hat{\mathbf{w}}_\ell^H \hat{\boldsymbol{\mathcal{H}}}^H \hat{\mathbf{R}}_{I+N}^{-1} \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_k \\ -\hat{\mathbf{w}}_k^T \hat{\boldsymbol{\mathcal{H}}}^T \hat{\mathbf{R}}_{I+N}^{-1*} \boldsymbol{\mathcal{H}}^* \hat{\mathbf{w}}_\ell^* \\ -\hat{\mathbf{w}}_\ell^T \hat{\boldsymbol{\mathcal{H}}}^T \hat{\mathbf{R}}_{I+N}^{-1*} \boldsymbol{\mathcal{H}}^* \hat{\mathbf{w}}_\ell^* \end{bmatrix} \qquad \mathbf{b}_{n+1} = \begin{bmatrix} \hat{\mathbf{w}}_k^H \hat{\boldsymbol{\mathcal{H}}}^H \hat{\mathbf{R}}_{I+N}^{-1} \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_\ell \\ \hat{\mathbf{w}}_\ell^H \hat{\boldsymbol{\mathcal{H}}}^H \hat{\mathbf{R}}_{I+N}^{-1} \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_\ell \\ \hat{\mathbf{w}}_k^T \hat{\boldsymbol{\mathcal{H}}}^T \boldsymbol{\mathcal{H}}^* \hat{\mathbf{R}}_{I+N}^{-1*} \hat{\mathbf{w}}_k^* \\ \hat{\mathbf{w}}_\ell^T \hat{\boldsymbol{\mathcal{H}}}^T \hat{\mathbf{R}}_{I+N}^{-1*} \boldsymbol{\mathcal{H}}^* \hat{\mathbf{w}}_k^* \end{bmatrix}$$

- cross-terms like $\hat{\mathbf{w}}_k^H \hat{\boldsymbol{\mathcal{H}}}^H \hat{\mathbf{R}}_{I+N}^{-1} \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_\ell$ are no longer zero, and "auto-terms" like $\hat{\mathbf{w}}_k^H \hat{\boldsymbol{\mathcal{H}}}^H \hat{\mathbf{R}}_{I+N}^{-1} \boldsymbol{\mathcal{H}} \hat{\mathbf{w}}_k$ are no longer real-valued
- $\mathbf{b}_{\mathrm{n}}$ and $\mathbf{b}_{\mathrm{n}+1}$ still orthogonal AND yield same SINR gain per symbol!!
- despite channel feedback, Rx must estimate 4 values comprising $\mathbf{b}_{\mathrm{n}}$

OUTLINE

- SISO Temporal Processing with interference pre-cancelling and "STBC" option
- MISO Space-Time Beamforming with interference pre-cancelling and STBC option
- MIMO Space-Only Beamforming with interference pre-cancelling and STBC option
- **MIMO Space-Time Beamforming with interference pre-cancelling & STBC option**
  - **work in progress!**